DECELERATION OF A PLASMOID IN A NONUNIFORM MAGNETIC FIELD

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1. In the channels of MHD devices many physical effects are associated with the nonuniformity of the flow and the magnetic field over the channel cross section. Investigation of the currents in a channel with nonconducting walls makes possible a complete study of the fringe effect associated with field nonuniformity and its dependence on the velocity profile and the magnetic field.

In [1] Shercliff investigated the effect of field nonuniformity on the induced emf resulting from the formation of fringe currents when the electrodes are insufficiently removed from the zones of field nonuniformity and showed that a step-type nonuniform magnetic field affects the emf at $x/h_1 < 3$, where x is the distance between the electrodes and the leading edge of the magnetic field and h_1 is a distance corresponding to half the channel height.

In [2] it was shown that the emf may differ from the theoretical value as a result of deceleration of the flow by the Lorentz force in the zone of interaction of the fringe current and the increasing magnetic field. Plasma deceleration in a nonuniform magnetic field was investigated in detail in [3] at an electron density $n_e \leq 10^{-14}$ cm^{-3}, and it was shown that such deceleration can be fairly important. When the flow is decelerated the plasma parameters change, which must be taken into account in MHD channels.

The development of MHD devices with magnetic Reynolds numbers $R_m \geq 1$ and magnetic fields nonuniform along the channel axis requires the experimental investigation of the relation between the characteristics of the device and the fringe effects in the field entrance and exit zones.

2. As a plasma source we used the discharge tube describedin [4]. The parameters of the plasma flow were as follows: velocity of the leading front of the plasmoid in a section 1 m away from the plasma source $v = 2.8 \cdot 10^6$ cm/sec, the plasma was almost completely singly ionized. The working medium was argon, the M number of the flow was 5, and the static pressure in the plasmoid was $p \leq 0.5$ kg/cm^2. The rectangular plastic channel measured 3×4 cm^2. The magnetic field was created by a Helmholtz coil system, into which a 900 μF capacitor band was discharged. The capacitor voltage varied from 0.5 to 3 kV depending on the magnetic field required. The coil diameter was 20 cm, and the distance between coils was 4 cm. The magnetic field in the gap at the center of the coils was 0.96 Wb/m^2 at a capacitor voltage of 3 kV and fell to zero at a distance of 10 cm from the center. The magnetic field was fairly uniform only on a distance of about 4 cm in the center of the magnet. The discharge half-period was $3 \cdot 10^{-3}$ sec, the time taken by the plasmoid to travel through the magnetic field was $\sim 1 \cdot 10^{-4}$ sec.

To synchronize the plasma flow with the magnetic field we used a time delay unit. The plasmoid entered the region of the magnetic field when the field reached its maximum value.

3. To investigate the emf induced by the motion of the plasma through an external transverse magnetic field we used tungsten electrodes 0.1 cm in diameter. The distance between the flat working surfaces varied from 4 cm, when the electrodes were mounted flush with the channel walls, to 0.5 cm, when each electrode projected 1.75 cm into the flow. The interelectrode gap was investigated every 0.5 cm. The magnetic field varied from 0.16 to 0.96 Wb/m^2. To measure the emf we connected to the electrodes an external resistance of 10^6 ohm, the signal from which was fed to the plates of an OK17M oscillograph. In calculating the emf, we took as the flow velocity the velocity of the leading front of the plasmoid before it entered the magnetic field, $v = 2.8 \cdot 10^6$ cm/sec.

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The dependence of the emf on the magnetic field for various interelectrode distances is shown in Fig. 1. The emf was determined from the oscillograms 10 μsec after the appearance of a signal at the resistance, at a point corresponding to the maximum.

In [5] in connection with the investigation of plasmoids obtained by means of discharge tubes it was shown that a considerable part of the plasmoid has a velocity equal to the velocity of the leading front. An experimental investigation showed that at any interelectrode distance (h) the emf varies nonlinearly with increase in the applied magnetic field and, for example, at h = 2 cm, B = 0.96 Wb/m^2, the experimental value of the emf is half the theoretical value. A similar picture is also observed at other interelectrode distances.

It is clear from Fig. 1 that the difference between the experimental and theoretical values of the emf increases with increase in the magnetic field.

In [1] it was shown that the measured potential difference between open electrodes may be less than the theoretical emf, if the point electrodes are located close to the zones of nonuniformity of the magnetic field. For a step field the sensitivity, i.e., the ratio of the measured to the theoretical emf, is less than unity at $x/h_1 \leq 3$. As x/h_1 decreases, the sensitivity falls sharply. If the magnetic field is curved, the sensitivity can be calculated from the following equation:

$$S = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \int_0^{\infty} \frac{B}{B_{\max}} \exp\left(\frac{-n\pi x}{2h_1}\right) d\left(\frac{x}{h_1}\right) \quad \left(S = \frac{U}{vBh}\right)$$

Here, S is the sensitivity, and n is an odd number.

Considering that

$$\sum_{n=1}^{\infty} \frac{1}{n} \exp\frac{-n\pi x}{2h_1} = \frac{1}{2} \ln \frac{1 + \exp(-\pi x / 2h_1)}{1 - \exp(-\pi x / 2h_1)}$$

and introducing the notation

$$z = x / h_1, f(z) = B / B_{\max}$$

we finally obtain

$$S = \frac{2}{\pi} \int_0^{\infty} [f(z) - 1] \ln \operatorname{cth} \frac{\pi z}{4} dz + \frac{2}{\pi} z \ln \operatorname{cth} \frac{\pi z}{2} \Big|_0^{\infty} + \frac{1}{2} \int_0^{\infty} \frac{z\, dz}{\operatorname{sh} {}^1/_4\, \pi z \operatorname{ch} {}^1/_4\, \pi z}$$

We calculated a series of values of S for z = 1-10. The integrals were evaluated in accordance with Simpson's rule (n = 20).

The sensitivity was calculated on the assumption of constancy of the velocity profile and the electrical conductivity over the channel cross section. In [6] it was shown that for large z ($z \gtrsim 1.9$) the sensitivities for homogeneous and Poiseuille velocity profiles almost coincide, the difference in sensitivity not exceeding 1%. Calculations showed that at $h_1 = 2$ cm, x = 10 cm, and z = 5, the sensitivity S = 0.98, i.e., there should not be any significant decrease in emf due to the fringe current.

In [1] to reduce emf losses, in the event that the electrodes were not sufficiently removed from the fringe current zones, it was proposed to introduce thin insulated baffles oriented along the flow into the channel in the zone of magnetic field nonuniformity. These baffles reduce the effectiveness of the fringe currents and thereby reduce charge losses. To verify the accuracy of the calculations plastic plates 0.1 cm thick with a sharp leading edge were introduced into the channel. Their length λ varied from 2 to 15 cm. The plate divided the channel into two parts of equal height and could be installed both in front of and behind the electrodes. The effect of the plate was investigated at only one interelectrode distance, h = 2 cm. When the plate was located in front of the electrodes, its trailing edge extended to the electrode axis, whereas when it was mounted behind the electrodes, the sharp leading edge touched the electrode axis.

The dependence of the emf on the magnetic field is shown in Fig. 2 for a channel with and without plates at an interelectrode distance h = 2 cm. It should be noted that in all the experiments the plasma parameters at the entrance to the magnetic field were the same ($v = 2.8 \cdot 10^6$ cm/sec, $\sigma \simeq 100$ mho/cm, M = 5). It was found that, irrespective of the length of the plate (λ = 2-15 cm), when it was located behind the electrodes the graphs of emf versus magnetic field coincided with each other and with the dependence $E = f(B)$ for the case without plates. The scatter of the points did not exceed 5%.

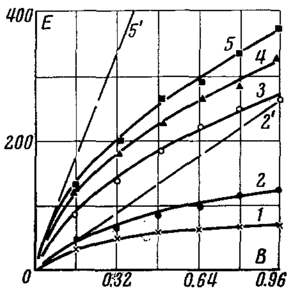

Fig. 1

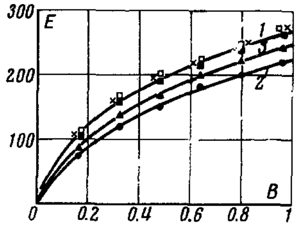

Fig. 2

Fig. 1. Electromotive force as a function of the transverse magnetic field for various interelectrode distances (h): 1) h = 0.5 cm; 2) h = 1 cm; 3) h = 2 cm; 4) h = 3 cm; 5) h = 4 cm; 2') theoretical emf for h = 1 cm; 5') for h = 4 cm.

Fig. 2. Electromotive force as a function of the magnetic field in the presence of plates in the channel: 1) without plates; 2) plates 2 cm long in front of electrodes; 3) plate 15 cm long in front of electrodes.

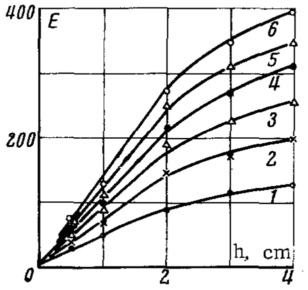

Fig. 3

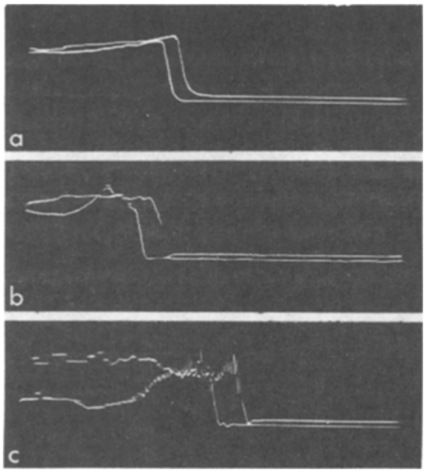

Fig. 4

Fig. 3. Electromotive force as a function of the interelectrode distance for various values of the transverse magnetic field: 1) B = 0.16 Wb/m^2; 2) B = 0.32 Wb/m^2; 3) B = 0.48 Wb/m^2; 4) B = 0.64 Wb/m^2; 5) B = 0.80 Wb/m^2; 6) B = 0.96 Wb/m^2.

Fig. 4. Oscillograms of probe measurements of the velocity of the leading front of the plasmoid as a function of the magnetic field (center of magnet): a) B = 0; b) B = 0.48 Wb/m^2; c) B = 0.96 Wb/m^2.

Thus, the experiments with a plate in the decay zone of the magnetic field confirm the calculations, i.e., for the given field and channel configuration the charge losses due to fringe currents are in fact small. Completely different results were obtained in the case of plates mounted in front of the electrodes. If the plate was short (λ = 2 cm), then the emf was less than in the case without plates (see Fig. 2, curve 2), which is attributable to the fact that a short plate introduced into a supersonic flow in the uniform field zone reduces the channel cross section for the passage of gas on account of the thickness of the plate and the boundary layer on it.

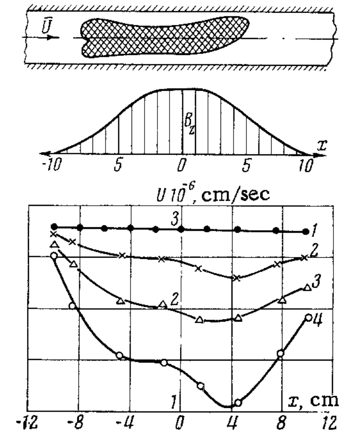

Fig. 5. Velocity of leading front of plasmoid as a function of the distance from the center of the magnetic field: a) channel; b) geometry of transverse magnetic field B; c) velocity of leading front as a function of distance from center of magnet: 1) B = 0; 2) B = 0.16 Wb/m^2; 3) B = 0.48 Wb/m^2; 4) B = 0.96 Wb/m^2.

Calculations show that the flow deceleration caused by the plate is 6-7%. The emf is chiefly affected by the discontinuity of the velocity profile near the channel axis due to the plate and the boundary layer. It was assumed that a plate 15 cm long in front of the electrodes completely intersects the region of nonuniform magnetic field. In this case the measured emf (curve 3 in Fig. 2) is greater than the emf for the case of a short plate. The calculations made above showed that the charge losses should not exceed a few percent. If the plate in front of the electrodes did not create additional hydrodynamic losses in the channel and did not disturb the velocity profile on the channel axis, then, according to the calculations, the measured emf would exceed 300 V at B = 0.96 Wb/m^2 and h = 2 cm, which is much greater than for the case of a channel without a plate.

The dependence of the emf on the interelectrode distance h is shown in Fig. 3 for various values of the magnetic field. It was found that at h > 2 cm the dependence E = f(h) becomes nonlinear. At h > 2 cm the end surfaces of the electrodes enter the boundary layer on the channel walls. At h ≤ 2 cm the emf varies linearly for all values of the magnetic field. At a channel height of 4 cm, when h = 2 cm or less each electrode projects 1 cm or more into the flow and is situated outside the boundary layer. By means of the graph in Fig. 3 we constructed the potential distribution over the height of the channel and the velocity profiles, since for a nonuniform velocity profile

$$B = \text{const}, \qquad E = B\int_0^h v\,dy$$

It was found that the velocity profile corresponds to a laminar flow and does not depend on the external magnetic field:

$$\frac{v}{v_\infty} = \left(\frac{y}{\delta}\right)^{1/2}$$

where δ is the thickness of the boundary layer and v_∞ is the velocity in the flow core. The absolute value of the velocity on the channel axis falls as the magnetic field increases. For example, if the velocity on the axis at B = 0.16 Wb/m^2 is taken as 1 ($v^0 = 1$), then at B = 0.48 Wb/m^2 $v^0 = 0.62$ and at B = 0.96 Wb/m^2 $v^0 = 0.45$, i.e., as the magnetic field increases by a factor of 6 the flow velocity on the channel axis is approximately halved. The deceleration of the flow is related with the Lorentz force that appears in the zone of magnetic field nonuniformity as a result of the interaction between the fringe current and the growing magnetic field, i.e., on account of the force $\mathbf{F} = c^{-1}\mathbf{j} \times \mathbf{B}$, where $\mathbf{j}$ is the fringe current density vector.

To explain the nondependence of the velocity profile on the external magnetic field we calculated the possibility of transition from Poiseuille flow to completely developed Hartmann flow on a magnetic field length of 10 cm.

The distance traveled by the plasma in the region of the magnetic field up to the establishment of a completely developed Hartmann flow, the so-called entrance length L, was determined from the equation L/h = R/H, where R is the Reynolds number for completely ionized argon, and H is the Hartmann number for the same conditions. It was found that transition from a Poiseuille to a Hartmann velocity profile at B = 1 Wb/m^2 requires an entrance length of about 10 m. Accordingly, under the conditions in question there was no appreciable deformation of the velocity profile.

4. The dependence of the velocity of the leading front of the plasmoid on the external magnetic field was studied experimentally. The velocity was measured with probes. Previous work on the simultaneous measurement of the velocity by means of probes and FEU-19 photomultipliers at the same points along the channel gave good agreement between the results. Steel probes 0.05 cm in diameter were introduced 1.5 cm into the flow on one side. The distance between the working surfaces of the probes was 1 cm. The entire lateral surface of the probes was covered with insulation. To each pair of probes in a plane perpendicular to the velocity vector we applied a voltage of 100 V from a 0.1-μF capacitor across an external resistance R = 1 kilohm. The signal from the external resistance of each pair of probes was fed to the plates of an OK17M oscillograph. The velocity of the leading front of the plasmoid was determined from the distance between the pairs of probes and the time taken by the leading front to traverse that distance measured on the oscillograms (Fig. 4).

Nine pairs of probes with an interval of 3 cm between pairs were mounted in the channel in order to cover the entire region of the magnetic field. Four pairs were located in front of the electrodes, i.e., in front of the center of the magnetic field, four pairs behind the center of the magnetic field, and one pair at the center of the field. As a check we measured the velocity with only two pairs of probes 3 cm apart, but displacing the magnet along the channel axis with an interval of 3 cm, in order to repeat the measurements at the same points but with less distortion of the flow by the probes.

In Fig. 5 the velocity of the leading front of the plasmoid is shown as a function of the magnetic field along the channel axis for values of the magnetic field at the center of the magnet equal to 0, 0.16, 0.48, and 0.96 Wb/m^2, respectively.

Clearly, the velocity of the leading front of the plasmoid falls depending on the distance from the center of the magnetic field, the first velocity jump beginning at a distance of 8 cm from the center of the magnet and ending at a distance of 5 cm from the center. Then comes a region with a small fall in velocity extending to the center of the magnet, and a sharp fall in velocity begins again on a channel length of 4 cm. The zone of the second velocity gradient is followed by an acceleration zone. The degree of acceleration of the leading front of the plasmoid is somewhat greater than the degree of deceleration in the region of growth of the magnetic field.

After several discharges a spot (see Fig. 5a), associated with the increase in the temperature and pressure of the plasma in the fringe current zones, forms on the lateral transparent walls of the channel, i.e., the walls perpendicular to the magnetic field. The spot is not symmetrical relative to the center of the magnet; it extends 8 cm upstream and 5 cm downstream. It should be noted that the zone of the first velocity gradient coincides with the beginning of the spot on the channel wall. The boundary of the spot and the measured velocity show that at these points the magnetic field interacts with the fringe currents in the zones of magnetic field nonuniformity.

The interval with acceleration of the leading front of the plasmoid indicates that in the given zone the magnetic field interacts with the fringe currents, the current density vector being opposite to the current density vector in the deceleration zones. The considerable acceleration may be associated with the fact that not all the plasmoid is accelerated to such an extent, but only a small part together with the leading front. It is clear from the graph that in the zone of constant magnetic field, i.e., for 2 cm on either side of the center of the magnet, the flow velocity is different. For example, at B = 0.96 Wb/m^2 for the sections x = −2 cm, x = 0, and x = 2 cm relative to the center of the magnet the flow velocities are, respectively, equal to

$$(1.5 \div 1.35 \div 1.2)\cdot 10^6 \text{ [cm/sec,]}$$

To check the accuracy of the probe measurements we successively set up the electrodes at the indicated points with h = 2 cm. We obtain the following values of the emf at B = 0.96 Wb/m^2: 290, 270, 230 V, which is quite close to the velocity ratio for the given magnetic field.

5. It was noted above that the introduction of a plate 15 cm long in front of the electrodes made it possible to increase the emf, although it was known by calculation that the charge losses due to the nearness of the fringe currents were inconsiderable.

To account for this effect we made probe measurements of the flow velocity over a plate 15 cm long in front of the center of the magnet as a function of the magnetic field. Five pairs of probes were mounted over the plate at a distance of 1.5 cm from the wall, again at intervals of 3 cm. It was found that the velocity of the leading front of the plasmoid falls at the same points more slowly in the presence than in the absence of a plate. For example, at points x = −1.5 cm upstream from the center of the magnet at B = 0.96 Wb/m^2 the velocities of the leading front were $1.5 \cdot 10^6$ and $1.8 \cdot 10^6$ cm/sec in the absence and in the presence of a plate, respectively, i.e., a plate in front of the electrodes, by dividing the fringe current into two loops, reduces its over-all effectiveness and hence the decelerating force.

Thus, in the channels of MHD devices with a nonuniform transverse magnetic field at $R_m \geq 1$ as a result of the interaction between the fringe currents and the magnetic field there will always be a deceleration of the flow that depends on the magnitude of the magnetic field, and to reduce the velocity losses in the channel it is necessary to introduce a series of plates in the magnetic field entrance zone so as to reduce the fringe currents, despite the additional gas-dynamic losses thus introduced.

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